



# Geostatistical History Matching coupled with Adaptive Stochastic Sampling: A zonation-based approach using Direct Sequential Simulation

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## A B S T R A C T

Advances in computing technology, over the past decades, allowed the development of a number of different History Matching (HM) techniques. Nevertheless, the simultaneous integration of production data under geological consistency, as part of the reservoir modelling workflow, still remains a challenge.

A geologically consistent approach aims to avoid solutions that are unrealistic under the reservoir's general geological characteristics. Unrealistic HM solutions often result in poor reservoir response forecasting. It is also essential to include only geologically realistic models for uncertainty assessment based on multiple models that are consistent with the geological data and also in order to match observed production history.

Geostatistical History Matching (GHM) can iteratively update static reservoir model properties through conditional assimilation constrained to the production data, using geologically consistent perturbation. Multiple stochastic realizations are assimilated following a zonation approach to account for the local match quality, providing a way to integrate regionalized discretization of parameters with production data and engineering knowledge.

The present project proposes a new HM technique applied in uncertain reservoir conditions represented by geologically consistent reservoir zonation, based on fault presence and production streamlines. The work explores the value of using a geologically consistent zonation associated with production wells in GHM regions, coupled with adaptive stochastic sampling and Bayesian inference for uncertainty quantification and optimization of geological and engineering properties. This novel approach makes use of the Direct Sequential Simulation (DSS) algorithm for generation of stochastic realizations and Particle Swarm Optimization (PSO) for parameter optimization. The approach is tested in a semi-synthetic case study based on a braided-river depositional environment.

**Keywords:** History Matching, Geostatistics, Direct Sequential Simulation, Uncertainty Quantification, Particle Swarm Optimization, Adaptive Stochastic Sampling.

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## 1. Introduction

Reservoir modelling is a crucial step in the development and management of petroleum reservoirs. An accurate reservoir model is one that honors all data at the scale and precision at which they are available. Information available to model the reservoir is continually updated over the course of field development. Integration of static and dynamic models is performed by modifying the static model in order to match the observed historical reservoir production data, using HM techniques.

However, the integration of both types of data into reservoir modelling is a challenging task, as the relationship between *hard data* and production data is highly non-linear. The process of HM tries to address this problem by applying changes to the reservoir model in order to minimize a given cost function, responsible for the quantification of the mismatch between the observed production data (historical data) and the dynamic model response (simulated data). This introduces another problem

associated with HM. Multiple reservoir models (static or dynamic) can produce equally matched responses, making HM an *ill-posed* problem.

Over recent years, several approaches have been developed to address History Matching of oil and gas reservoirs. Methods depending on data assimilation, like the Ensemble Kalman Filter (Evensen, *et al.* 2007), gradual deformation (Hu, *et al.* 2001), probability perturbation (Caers & Hoffman, 2006) or stochastic sequential simulation and co-simulation (Mata-Lima, 2008), (Le Ravalec-Dupin & Da Veiga, 2011) have been proposed.

Other methods like Stochastic Optimisation algorithms, allow reducing computational times by sampling from the ensemble of best matched models in order to improve matches. Algorithms such as the Neighbourhood Algorithm (Sambridge, 1999), the Genetic Algorithm (Erbas & Christie, 2007), the Particle Swarm Optimisation (Mohamed, 2011) and Differential Evolution (Hajidazeh, *et al.* 2009) have been developed on recent years. By sampling only for the models that fit better according to the evolutionary principle, adaptive stochastic

optimisation allows a reduction in computational costs.

The present project proposes a new HM technique, coupling Traditional GHM methodologies with the integration of Adaptive Stochastic Sampling. The proposed methodology applies a GHM technique under uncertain reservoir conditions represented by geologically consistent reservoir zonation, based on fault presence and fluid production streamlines. The work explores the value of using a geologically consistent zonation methodology, associated with production wells in Geostatistical History Matched regions, coupled with adaptive stochastic sampling and Bayesian inference for uncertainty quantification and optimization of geological and engineering properties.

This novel approach makes use of the Direct Sequential Simulation (DSS) (Soares, 2001) algorithm for generation of stochastic realizations and Particle Swarm Optimization (PSO) (Kennedy & Eberhart, 1995) for parameter optimization. The approach is tested in a semi-synthetic case study based on a braided-river depositional environment.

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## 2. Methodology Workflow

Following the works on regionalization-based HM algorithms done by Hoffman & Caers (2005) and Mata-Lima (2008), the proposed methodology couples adaptive stochastic sampling with Zonation-Based GHM. The proposed methodology is synthesized by the following workflow:

### First step (Zonation-Based GHM; Fig. 1):

1. Regionalization of the reservoir area according to a given fault model and area of influence of the wells, resulting in a cube with a zone being assigned for each well or for a group of wells.

2. Simulation of a set of permeability ( $k$ ) and porosity ( $\phi$ ) realizations through DSS, honouring the well data, histograms and spatial distribution revealed by the variogram;

3. Evaluation of the dynamic responses for each of the realizations and calculation of the

mismatch between the simulated response and real production data, using an objective function;

4. Calculation of a correlation coefficient between each dynamic response and generation of a cube composed of different correlation coefficients per zone;

5. Creation of a composite cube of  $\phi$  and  $k$ , with each region being populated by the corresponding realization with the lowest mismatch, calculated in step 3;

6. Return to step 2, using Co-DSS and the cubes calculated in steps 4 and 5 as local correlation coefficient and soft data, respectively. The algorithm is expected to run up to a maximum number of iterations, or until a pre-defined mismatch value is reached.

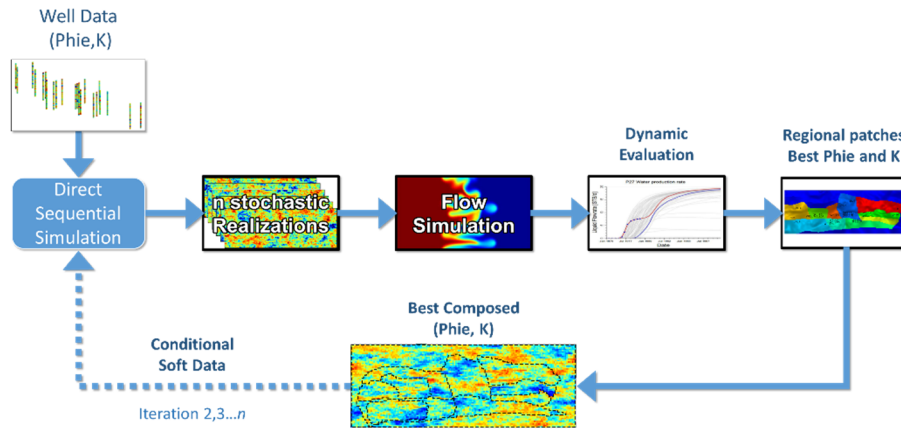


Fig. 1. Zonation-Based HM Workflow (First step)

## Second step (Coupling with Adaptive Stochastic Sampling; Fig. 2):

The second stage can be considered as an outer loop, encompassing the results obtained by the first step. At each iteration of the Zonation-Based GHM loop, a best  $\phi$  and  $k$  cube is found, along with their respecting correlation coefficient cubes. A process of matching relevant engineering and geological parameters will then occur, using Bayesian inference

and Adaptive Stochastic Sampling. The second phase runs for a maximum number of iterations.

For this work, the parameters considered for perturbation were the variogram parameters being used to generate the geostatistical realizations on the inner GHM loop, histogram perturbation and fault transmissibilities. Other perturbation parameters, geological or engineering, can be used with the same proposed procedure.

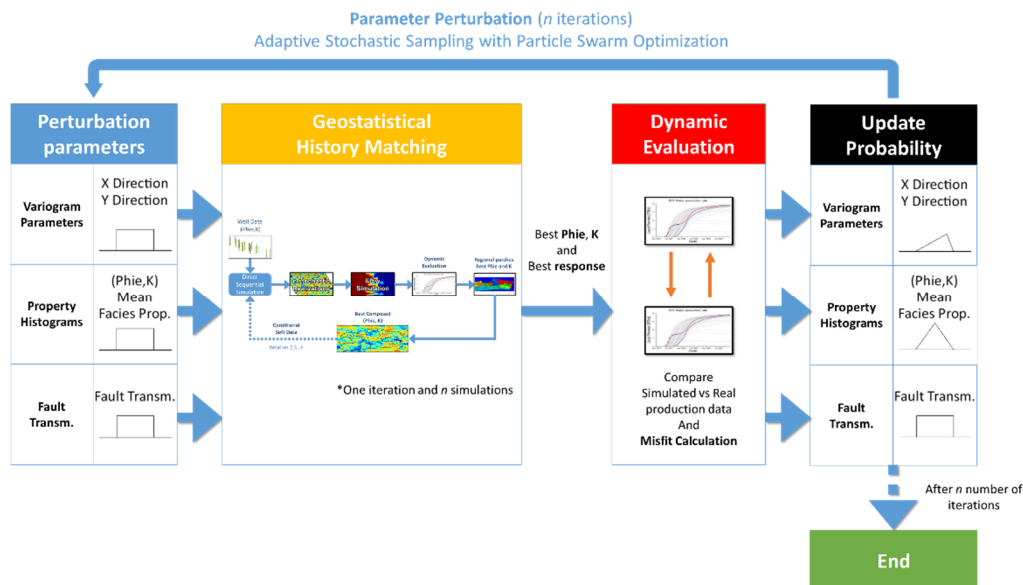


Fig. 2. Coupling with Adaptive Stochastic Sampling. (Second step) GHM is shown in detail in Fig. 1.

## 2.1. Dynamic evaluation

### 2.1.1. Objective Function

The objective function is formulated as a mathematical expression that measures how close a problem solution (simulated data) is towards an optimal value (observed data). The definition of such a metric is critical in order to achieve convergence in the iterative procedure. The most commonly used objective function for HM is the least square norm,

which calculates a measure of the discrepancy between the simulated and observed value, returning a “mismatch” value,  $M$ . The least squares norm formula to be used in the present work is defined as:

$$M = \min \sum_{t=i}^n \frac{(R_{t,obs} - R_{t,sim})^2}{2\sigma^2} \quad (1)$$

where  $R_{t,obs}$  and  $R_{t,sim}$  are, respectively, the observed and simulated values of a given variable at

timestep  $t$ .  $\sigma^2$  is the error, or standard deviation, associated to the measurement of a the same variable at the same timestep.

### 2.1.2. Correlation coefficient

Due to the importance of the correlation coefficient in the proposed GHM workflow, it is crucial to make sure its value mimics the misfit value obtained by the objective function. Meaning, a process that integrates calculations of misfit and

correlation coefficients should guarantee that when calculated, these values change symmetrically and in the same proportion, under different configurations of observation data curves. This issue was also tackled, within the scope of this work, with the development of a new approach to calculate a correlation coefficient based on the misfit value. The following step-by-step list describes the calculation of the correlation coefficient being used by the GHM algorithm.

1. For all timesteps and for the production variable to be matched, calculate a response difference, and the corresponding error, respectively  $\Delta_{sim}$  and  $\Delta_{sigma}$ , according to the following:

$$\Delta_{sim} = |R_{(t,obs)} - R_{(t,sim)}| \quad (2)$$

$$\Delta_{sigma} = \sigma_t^2 - \Delta_{sim} \quad (3)$$

2. Calculate the final deviation towards observed data for a given timestep,  $\Delta_{obs}$ , according to:

$$\Delta_{obs} = \begin{cases} R_{t,obs} - (\Delta_{sim} + \Delta_{sigma}) & \text{if } R_{t,obs} \geq \Delta_{sim} + \Delta_{sigma} \\ 0 & \text{if } R_{t,obs} < \Delta_{sim} + \Delta_{sigma} \end{cases} \quad (4)$$

3. Normalization of the value obtained in step 2 according to the following:

$$x_t = \begin{cases} 1 - \frac{\Delta_{sim}}{(\Delta_{sim} + \Delta_{sigma} + \Delta_{obs})} & \text{if } \Delta_{sim} \leq \Delta_{sigma} + \Delta_{obs} \\ 0 & \text{if } \Delta_{sim} > \Delta_{sigma} + \Delta_{obs} \end{cases} \quad (5)$$

4. The final correlation coefficient  $\bar{x}$  for a given response is obtained by:

$$\bar{x} = \frac{\sum_{t=1}^N x_t}{N} \quad (6)$$

## 2.2. Zonation-Based GHM

Zonation methodologies (also known as regionalization or compartmentalization) are a common approach in reservoir engineering and HM to reduce the number of parameters of a given reservoir description. One of the objectives of this

work was to explore different regionalization methodologies and their influence on the HM quality and convergence. The purpose behind this study was to arrive at a better zonation methodology to apply to the coupling stage. Fig. 3 is a presentation of the explored zonation methodologies.

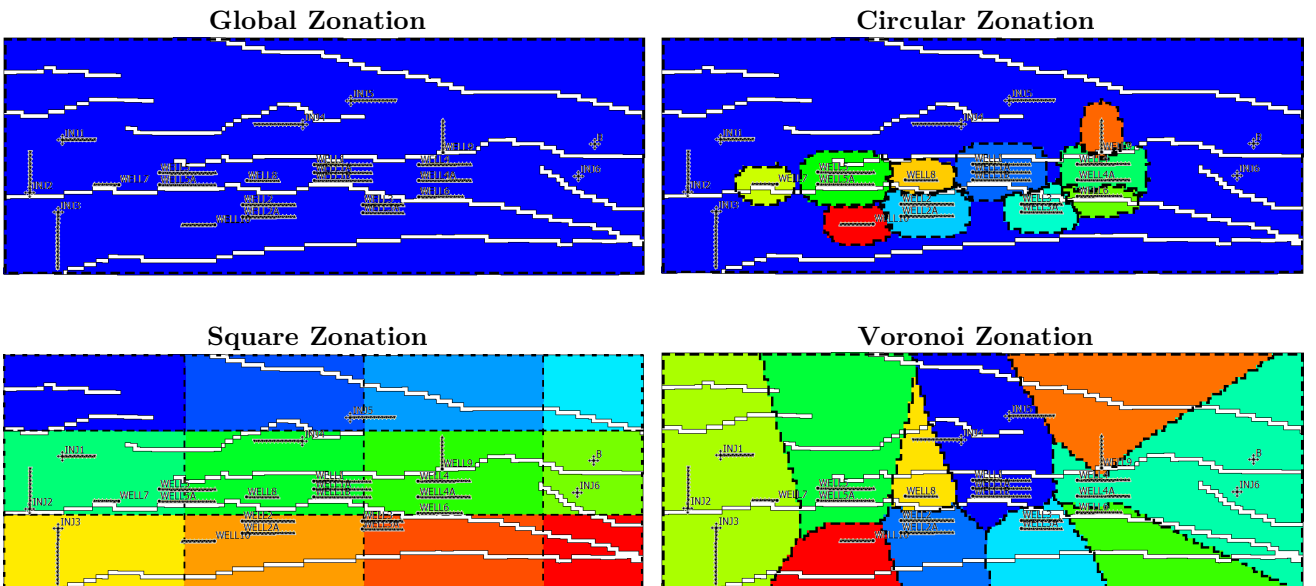


Fig. 3. Explored Zonation methodologies

### 2.2.1. Fault and Streamline-based Zonation

After observing the results obtained from the running of several GHM loop using the regionalization patterns described in Chapter 2.2 (the reader is encouraged to the main document for a view of the results), a hybrid like regionalization pattern was adopted in order to capture the accuracy obtained by the different regionalization methods.

For this, a regionalization method containing aspects of geological consistency, fluid flow paths along with their area of influence and fault zonation, was proposed. This regionalization method was primarily considered to improve mismatch results on the GHM stage, while maintaining a degree of geologic consistency during parameter perturbation. Fig. 4 illustrates the selected regionalization pattern to be used in the coupling stage of the algorithm.

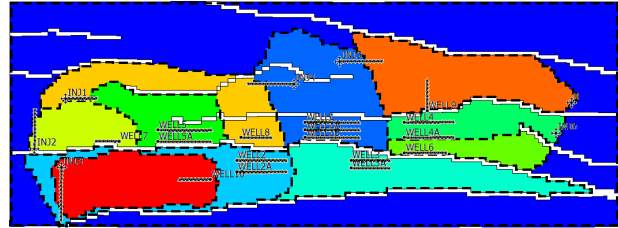


Fig. 4. Fault and Streamline-based Zonation

Within the scope of this work, uncertainty was assumed to exist in the horizontal and vertical ranges and on the composition of *hard data* histograms for porosity and permeability. Uncertainty was also assumed on fault transmissibilities of the model. Tab. 1 summarizes the selection of prior distribution ranges for the coupling stage of the algorithm.

Tab. 1. Selection of prior distribution ranges.

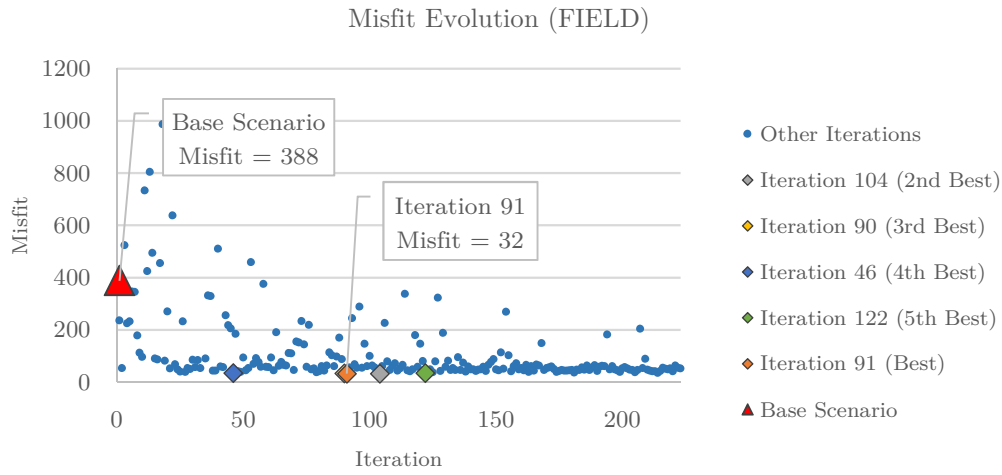
Property	Parameter	Variable Name	Distribution Type	Prior Range
Porosity	Horizontal. Range	$\$pororange1$	Discrete Uniform	[40, 80]
	Vertical. Range	$\$pororange2$	Discrete Uniform	[10, 30]
	Facies 2 Proportion	$\$poro\_fac\_2$	Uniform	[0.0, 0.3]
	Facies 1 Mean	$\$poro\_mean\_1$	Uniform	[0.15, 0.19]
	Facies 2 Mean	$\$poro\_mean\_2$	Uniform	[0.19, 0.23]
Permeability	Horizontal. Range	$\$permrange1$	Discrete Uniform	[40, 80]
	Vertical. Range	$\$permrange2$	Discrete Uniform	[10, 30]
	Facies 2 Proportion	$\$perm\_fac\_2$	Uniform	[0.7, 1.0]
	Facies 1 Mean	$\$perm\_mean\_1$	Uniform	[-1.5, 1.5]
	Facies 2 Mean	$\$perm\_mean\_2$	Uniform	[1.5, 3.0]
Fault Transm.	All Fault Transm.	$\$fttrans (2,3,5,6,7,8,9)$	Uniform	[0.0, 1.0]

## 3. Results and discussion

### 3.1. Coupling with Adaptive Stochastic Sampling

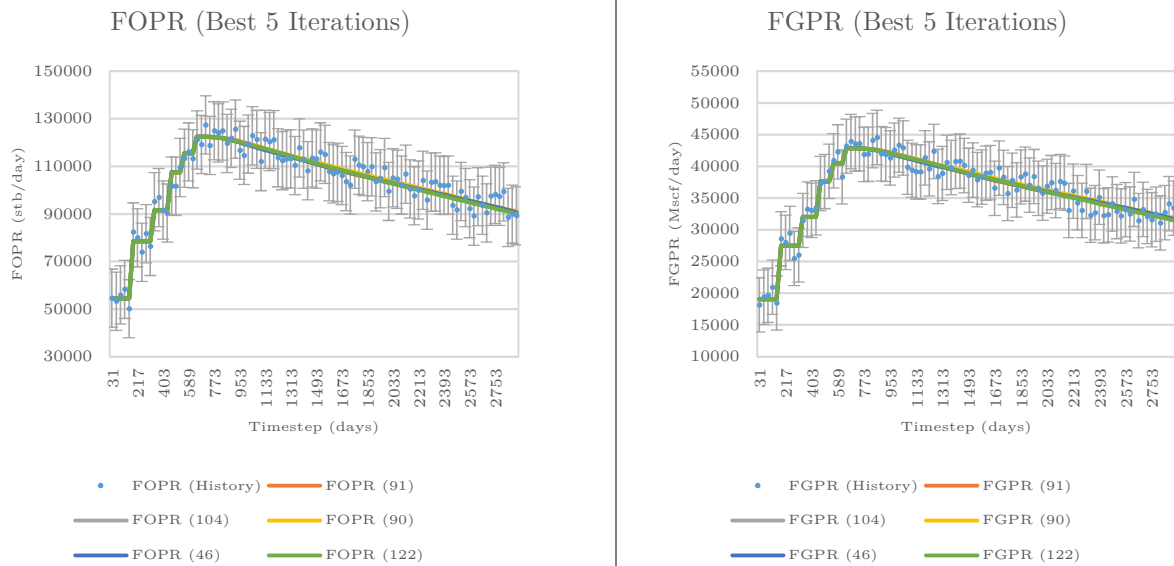
As previously mentioned on Chapter 2.2.1, for the coupling of GHM with Adaptive Stochastic sampling, fault and streamline based zonation was selected to be integrated on the GHM loop. The inner zonation-based GHM loop was set to run 5 simulations at every iteration. For the Adaptive

Stochastic Sampling outer loop, Particle Swarm Optimization was selected to optimize the perturbed parameters depicted on Tab. 1, over a total of 223 iterations. A total of 1115 fluid flow simulations were ran. From the observation of the misfit evolutions for FIELD data (Fig. 5), it is possible to interpret that convergence occurred at around iteration 100, when 4 of the 5 lowest misfits were obtained.

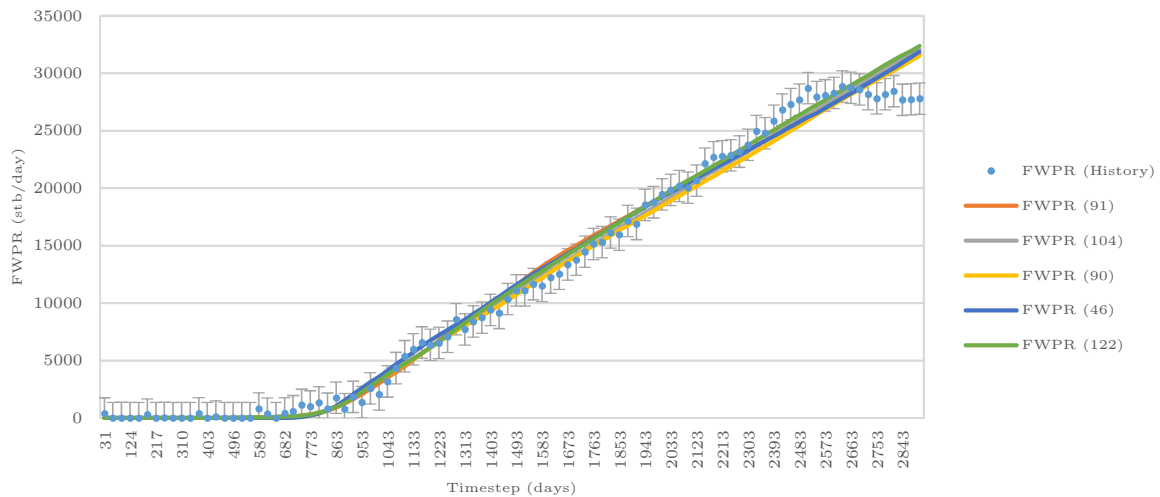


**Fig. 5.** Misfit Evolution of the Zonation-based GHM, coupled with Adaptive Stochastic Sampling

Fig. 6 shows the fluid flow response for the best 5 iterations of the run.



FWPR (Best 5 Iterations)



**Fig. 6.** Fluid flow response for FOPR (Top Left), FGPR (Top Right) and FWPR (Bottom) for GHM coupled with Adaptive Stochastic Sampling

Fig. 7 shows the Parameter vs Misfit plots for the perturbation of fault transmissibilities. On some of the parameters, a dispersion of values can be observed, along the lower values of misfit. However, it is possible to interpret the concentration of most

frequent parameter values for lower misfit iterations, namely on faults 3, 5, 6, with remaining fault transmissibility values being too sparse to assign a specific interval with a degree of confidence.

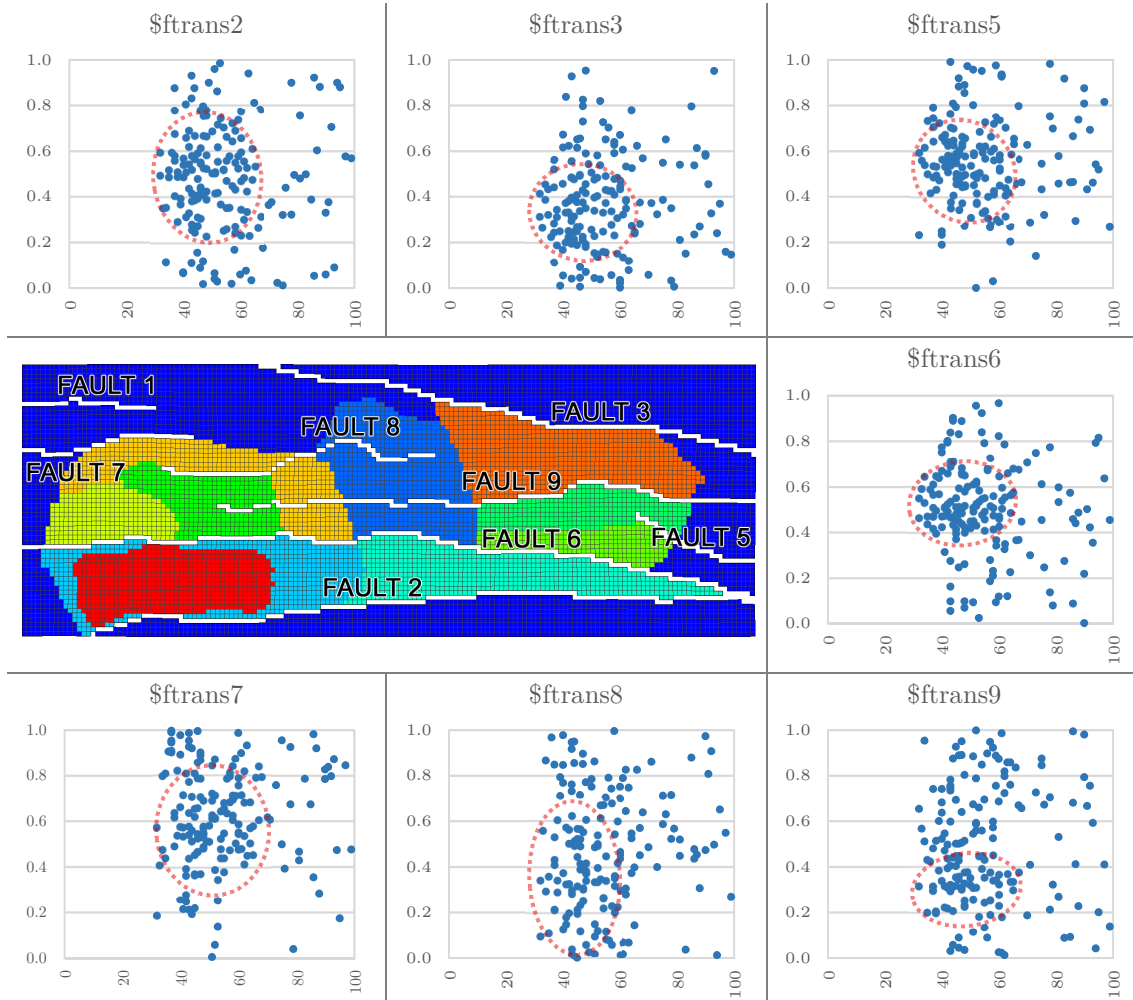


Fig. 7. Fault Transmissibility parameter value (y axis) vs Misfit (x axis)

Fig. 8 shows the concentration of parameter values for horizontal and vertical permeability ranges over the course of the iterations. A concentration of lower values for horizontal

permeability ranges ( $\$permrange1$ ) is observed, while for vertical ranges, the spread of parameter values, within the assumed prior range, corresponding to lower misfits, is much higher.

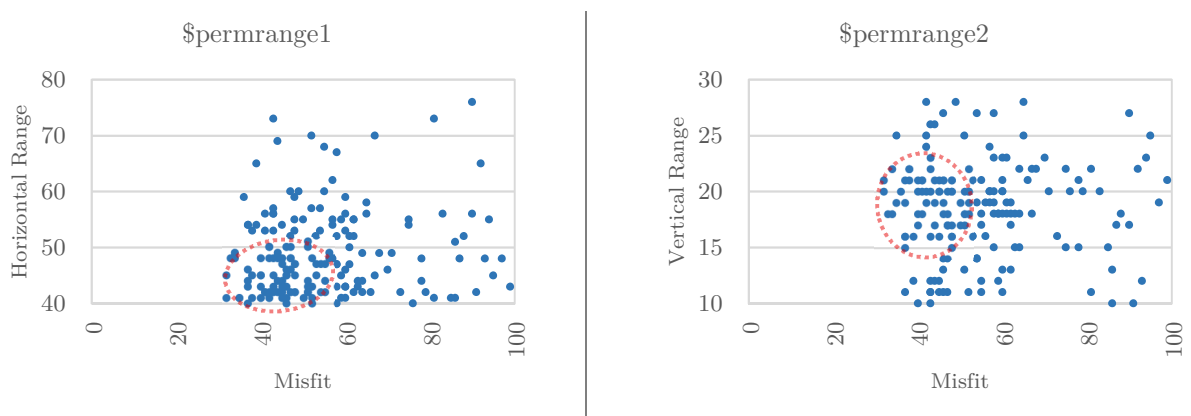
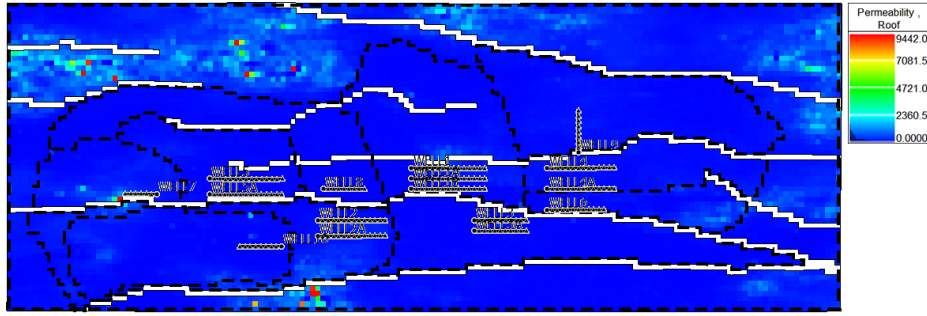


Fig. 8. Permeability Range vs Misfit (Left – Horizontal Range, Right – Vertical range)



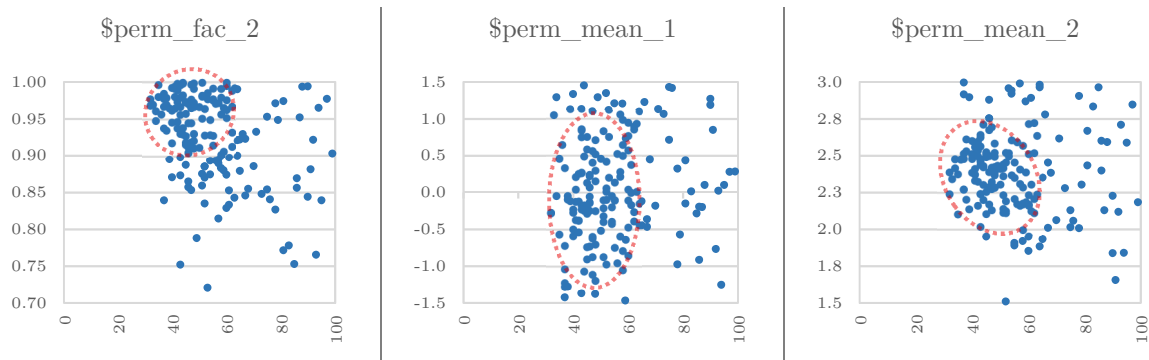
Fig. 9 shows a top view of the Permeability field for the best simulation (Iteration 91, simulation 5).



**Fig. 9.** Top view of the Best Permeability realization (Iteration 91, Simulation 5)

Regarding histogram perturbation for permeability, Fig. 10 shows the concentration of parameter values obtained for histogram perturbation, over the course of the iterations. There is a concentration of values for the proportion of Permeability Facies 2 occurring at higher values of the assumed prior distribution range. Permeability means for Facies 1 does not display an identifiable

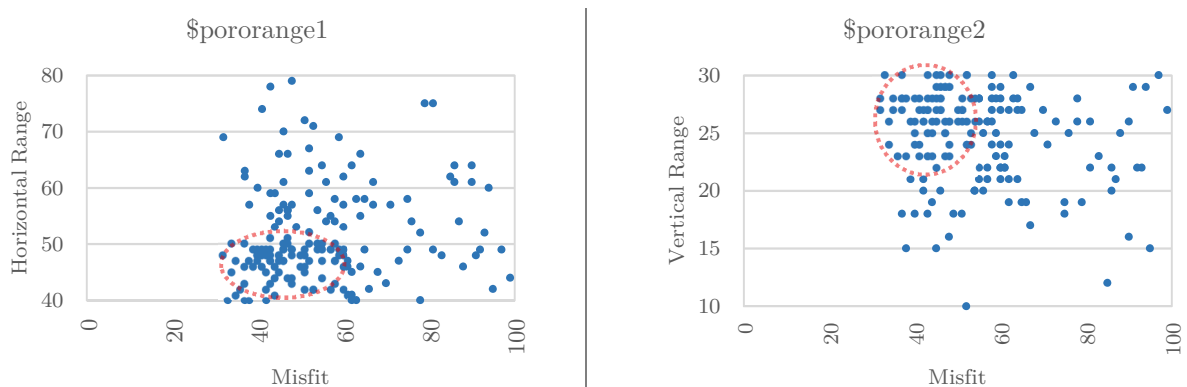
pattern, meaning that probably the perturbation for this parameter does not have a relevant enough effect on dynamic response, for the sampling algorithm to detect. As for Permeability means for facies 2, a concentration of parameter values at the top half part of the parameter distribution interval, is observed, on values ranging from  $2.4 \pm 0.3$ .



**Fig. 10.** Permeability Histogram Perturbation (y axis – parameter value) vs Misfit (x axis), (Left – Facies 2 Proportion, Middle – Facies 1 Mean, Right – Facies 2 Mean)

For variogram parameter perturbation of porosity, Fig. 11 shows the concentration of parameter values for horizontal and vertical porosity ranges over the course of the iterations. A concentration of lower values for horizontal porosity

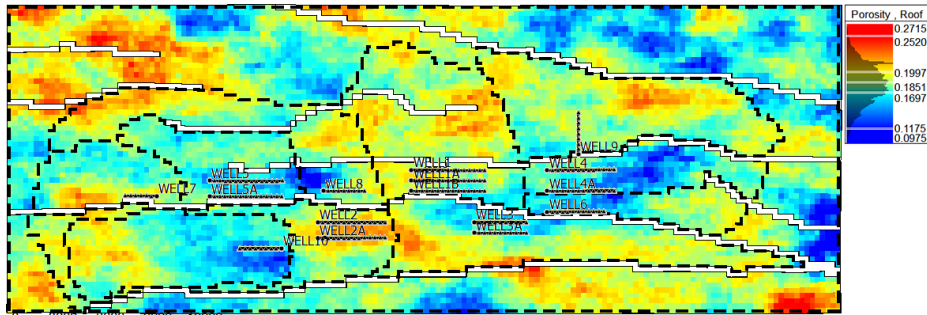
ranges ( $\$pororange1$ ) can be observed, while for vertical ranges, the concentration tends towards higher values of the adopted perturbation distribution.



**Fig. 11.** Porosity Range vs Misfit (Left – Horizontal Range, Right – Vertical range)



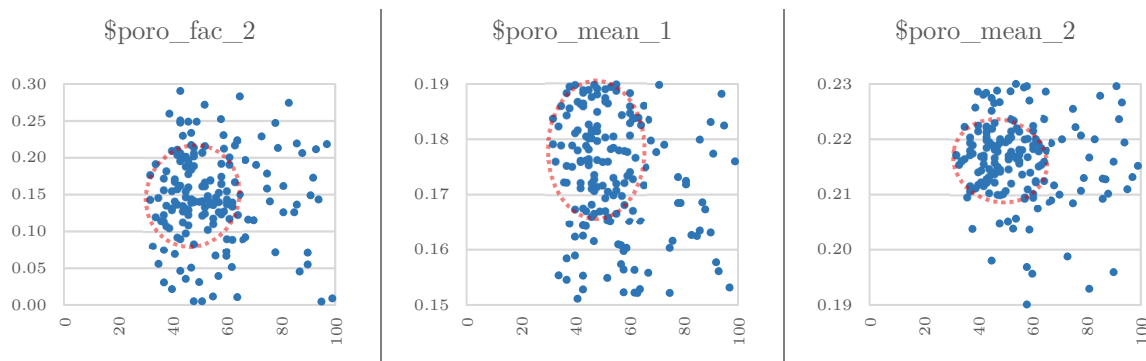
Fig. 12 shows a top view of the Permeability field for the best simulation (Iteration 91, Simulation 5).



**Fig. 12.** Top view of the Best Porosity realization (Iteration 91, Simulation 5)

Regarding histogram perturbation for porosity, Fig. 13 shows the concentration of parameter values obtained, over the course of the iterations. There is a concentration of values for Porosity Facies 2 proportions occurring at the mid-region of the prior

distribution interval. Porosity means for Facies 1 converges towards values around  $0.15 \pm 0.05$ , while Porosity means for facies 2, shows a concentration of parameter values at the top half part of the interval, for values between 0.21 and 0.22.



**Fig. 13.** Porosity Histogram Perturbation (y axis – parameter value) vs Misfit (x axis) (Left – Facies 2 Proportion, Middle – Facies 1 Mean, Right – Facies 2 Mean)

## 4. Conclusions and Future Work

The motivation behind the presented work was to contribute with an integrated workflow regarding GHM, by proposing an algorithm capable of coupling a traditional GHM method with Adaptive Stochastic Sampling, addressing the local match of production data, under a zonation-based methodology. A workflow was presented, showing very positive and promising results with a possibility to be extended and applied to other case studies, different parametrization selections or alternate regionalization methodologies.

The advantages of the proposed methodology are:

- Addressing perturbation in a geologically consistent manner;
- Respecting fluid production flow pattern, by usage of production streamlines;

- Reduced parametrization of static properties with the discretization of the reservoir;
- Ability to apply perturbation of static and engineering parameters and quantifying their uncertainty, while integrating a traditional GHM process;
- Ability to provide better results than other standard regionalization methods, under less time;
- Ability to generate multiple matched models that can be used to forecast production.

The application of the proposed methodology showed promising results, with the achieving of

multiple History Matched models with considerably low misfits (lowest misfit of 32, on a 96 timestep run). The creation of an ensemble of multiple history matched models by solving the *ill-posed* calibration problem, allows the prediction of reservoir behavior with a degree of uncertainty. Nevertheless, a limitation of the proposed methodology is its dependency on the right choice of prior ranges, which is essential for good optimization results.

The successful application of the proposed methodology to a challenging semi-synthetic reservoir case study delivers good perspectives for its application to real cases. Further research on this area could be focused towards performing forecasting using the proposed methodology, applying other types of parameter perturbation, alternate methods of reservoir zonation, integration of the methodology with facies perturbation or seismic inversion.

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